THE MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

24th Annual

AMC 8

(American Mathematics Contest 8)

Tuesday, November 18, 2008

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

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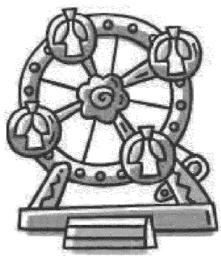
1. Susan had \$50 to spend at the carnival. She spent \$12 on food and twice as much on rides. How many dollars did she have left to spend?



(A) 12 **(B)** 14 **(C)** 26 **(D)** 38 **(E)** 50

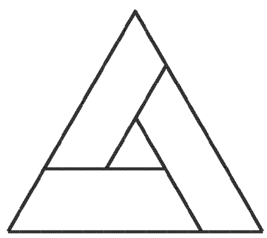
2. The ten-letter code BEST OF LUCK represents the ten digits 0–9, in order. What 4-digit number is represented by the code word CLUE?

(A) 8671 **(B)** 8672 **(C)** 9781 **(D)** 9782 **(E)** 9872

3. If February is a month that contains Friday the 13^{th}, what day of the week is February 1 ?

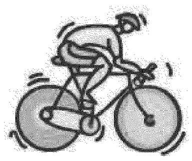
(A) Sunday **(B)** Monday **(C)** Wednesday **(D)** Thursday **(E)** Saturday

4. In the figure, the outer equilateral triangle has area 16, the inner equilateral triangle has area 1, and the three trapezoids are congruent. What is the area of one of the trapezoids?



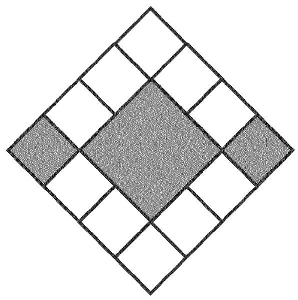
(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

5. Barney Schwinn notices that the odometer on his bicycle reads 1441, a palindrome, because it reads the same forward and backward. After riding 4 more hours that day and 6 the next, he notices that the odometer shows another palindrome, 1661. What was his average speed in miles per hour?



(A) 15 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 22

6. In the figure, what is the ratio of the area of the gray squares to the area of the white squares?

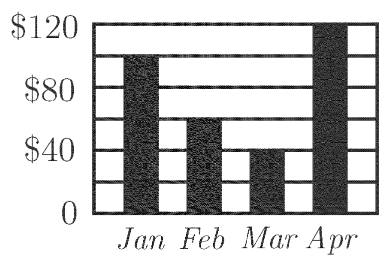


(A) 3 : 10 **(B)** 3 : 8 **(C)** 3 : 7 **(D)** 3 : 5 **(E)** 1 : 1

7. If $\frac{3}{5} = \frac{M}{45} = \frac{60}{N}$, what is $M + N$?

(A) 27 **(B)** 29 **(C)** 45 **(D)** 105 **(E)** 127

8. Candy sales of the Boosters Club for January through April are shown. What were the average sales per month in dollars?



(A) 60 **(B)** 70 **(C)** 75 **(D)** 80 **(E)** 85

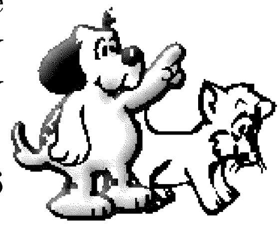
9. In 2005 Tycoon Tammy invested \$100 for two years. During the first year her investment suffered a 15% loss, but during the second year the remaining investment showed a 20% gain. Over the two-year period, what was the change in Tammy's investment?

(A) 5% loss **(B)** 2% loss **(C)** 1% gain **(D)** 2% gain **(E)** 5% gain

10. The average age of the 6 people in Room A is 40. The average age of the 4 people in Room B is 25. If the two groups are combined, what is the average age of all the people?

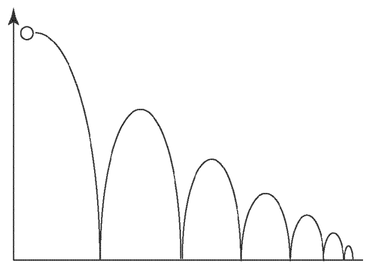
(A) 32.5 **(B)** 33 **(C)** 33.5 **(D)** 34 **(E)** 35

11. Each of the 39 students in the eighth grade at Lincoln Middle School has one dog or one cat or both a dog and a cat. Twenty students have a dog and 26 students have a cat. How many students have both a dog and a cat?



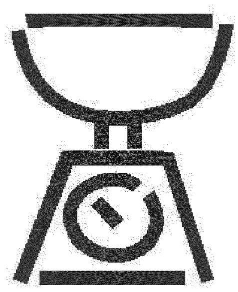
(A) 7 **(B)** 13 **(C)** 19 **(D)** 39 **(E)** 46

12. A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5 meters?



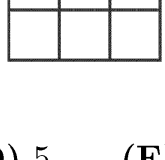
(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

13. Mr. Harman needs to know the combined weight in pounds of three boxes he wants to mail. However, the only available scale is not accurate for weights less than 100 pounds or more than 150 pounds. So the boxes are weighed in pairs in every possible way. The results are 122, 125 and 127 pounds. What is the combined weight in pounds of the three boxes?



(A) 160 **(B)** 170 **(C)** 187 **(D)** 195 **(E)** 354

14. Three As, three Bs and three Cs are placed in the nine spaces so that each row and column contain one of each letter. If A is placed in the upper left corner, how many arrangements are possible?

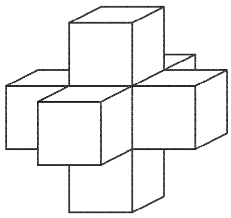


(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

15. In Theresa's first 8 basketball games, she scored 7, 4, 3, 6, 8, 3, 1 and 5 points. In her ninth game, she scored fewer than 10 points and her points-per-game average for the nine games was an integer. Similarly in her tenth game, she scored fewer than 10 points and her points-per-game average for the 10 games was also an integer. What is the product of the number of points she scored in the ninth and tenth games?

(A) 35 **(B)** 40 **(C)** 48 **(D)** 56 **(E)** 72

16. A shape is created by joining seven unit cubes, as shown. What is the ratio of the volume in cubic units to the surface area in square units?

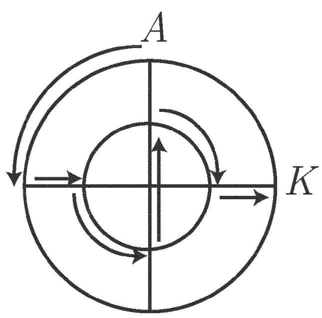


(A) $1:6$ **(B)** $7:36$ **(C)** $1:5$ **(D)** $7:30$ **(E)** $6:25$

17. Ms. Osborne asks each student in her class to draw a rectangle with integer side lengths and a perimeter of 50 units. All of her students calculate the area of the rectangle they draw. What is the difference between the largest and smallest possible areas of the rectangles?

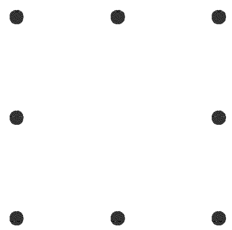
(A) 76 **(B)** 120 **(C)** 128 **(D)** 132 **(E)** 136

18. Two circles that share the same center have radii 10 meters and 20 meters. An aardvark runs along the path shown, starting at A and ending at K. How many meters does the aardvark run?



(A) $10\pi+20$ **(B)** $10\pi+30$ **(C)** $10\pi+40$ **(D)** $20\pi+20$ **(E)** $20\pi+40$

19. Eight points are spaced at intervals of one unit around a 2×2 square, as shown. Two of the 8 points are chosen at random. What is the probability that the points are one unit apart?

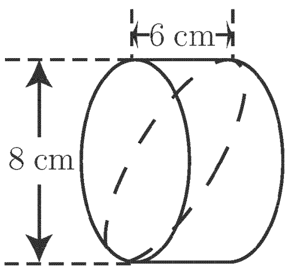


(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{4}{11}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{4}{7}$

20. The students in Mr. Neatkin's class took a penmanship test. Two-thirds of the boys and $\frac{3}{4}$ of the girls passed the test, and an equal number of boys and girls passed the test. What is the minimum possible number of students in the class?

(A) 12 **(B)** 17 **(C)** 24 **(D)** 27 **(E)** 36

21. Jerry cuts a wedge from a 6-cm cylinder of bologna as shown by the dashed curve. Which answer choice is closest to the volume of his wedge in cubic centimeters?

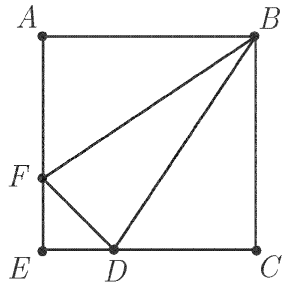


(A) 48 **(B)** 75 **(C)** 151 **(D)** 192 **(E)** 603

22. For how many positive integer values of n are both $\frac{n}{3}$ and $3n$ three-digit whole numbers?

(A) 12 **(B)** 21 **(C)** 27 **(D)** 33 **(E)** 34

23. In square $ABCE$, $AF = 2FE$ and $CD = 2DE$. What is the ratio of the area of $\triangle BFD$ to the area of square $ABCE$?



(A) $\frac{1}{6}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{7}{20}$

24. Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{11}{60}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{7}{30}$

25. Margie's winning art design is shown. The smallest circle has radius 2 inches, with each successive circle's radius increasing by 2 inches. Approximately what percent of the design is black?

(A) 42 **(B)** 44 **(C)** 45 **(D)** 46 **(E)** 48

2008

AMC 8

DO NOT OPEN UNTIL TUESDAY, NOVEMBER 18, 2008

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE NOVEMBER 18, 2008. Nothing is needed from inside this package until November 18.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.

The MATHEMATICAL ASSOCIATION OF AMERICA
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24th Annual

AMC 8

(American Mathematics Contest 8)

Solutions Pamphlet

Tuesday, NOVEMBER 18, 2008

This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students. However, the publication, reproduction, or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. *Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

Correspondence about the problems and solutions should be addressed to:

Ms. Bonnie Leitch, AMC 8 Chair / bleitch@earthlink.net
548 Hill Avenue, New Braunfels, TX 78130

Orders for prior year Exam questions and Solutions Pamphlets should be addressed to:

Attn: Publications
American Mathematics Competitions
University of Nebraska-Lincoln
P.O. Box 81606
Lincoln, NE 68501-1606

1. **Answer (B):** Susan spent $2 \times 12 = \$24$ on rides, so she had $50 - 12 - 24 = \$14$ to spend.

2. **Answer (A):** Because the key to the code starts with zero, all the letters represent numbers that are one less than their position. Using the key, C is $9 - 1 = 8$, and similarly L is 6, U is 7, and E is 1.

BEST	OF	LUCK
0 1 2 3	4 5	6 7 8 9

   CLUE = 8671

3. **Answer (A):** A week before the 13^{th} is the 6^{th}, which is the first Friday of the month. Counting back from that, the 5^{th} is a Thursday, the 4^{th} is a Wednesday, the 3^{rd} is a Tuesday, the 2^{nd} is a Monday, and the 1^{st} is a Sunday.

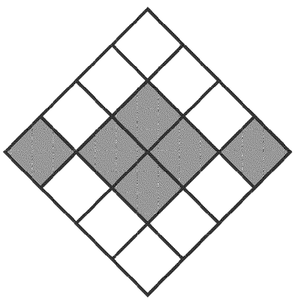
   OR

   Counting forward by sevens, February 1 occurs on the same day of the week as February 8 and February 15. Because February 13 is a Friday, February 15 is a Sunday, and so is February 1.

4. **Answer (C):** The area of the outer triangle with the inner triangle removed is $16 - 1 = 15$, the total area of the three congruent trapezoids. Each trapezoid has area $\frac{15}{3} = 5$.

5. **Answer (E):** Barney rides $1661 - 1441 = 220$ miles in 10 hours, so his average speed is $\frac{220}{10} = 22$ miles per hour.

6. **Answer (D):** After subdividing the central gray square as shown, 6 of the 16 congruent squares are gray and 10 are white. Therefore, the ratio of the area of the gray squares to the area of the white squares is $6 : 10$ or $3 : 5$.



7. **Answer (E):** Note that $\frac{M}{45} = \frac{3}{5} = \frac{3\cdot9}{5\cdot9} = \frac{27}{45}$, so $M = 27$. Similarly, $\frac{60}{N} = \frac{3}{5} = \frac{3\cdot20}{5\cdot20} = \frac{60}{100}$, so $N = 100$. The sum $M + N = 27 + 100 = 127$.

OR

Note that $\frac{M}{45} = \frac{3}{5}$, so $M = \frac{3}{5} \cdot 45 = 27$. Also $\frac{60}{N} = \frac{3}{5}$, so $\frac{N}{60} = \frac{5}{3}$, and $N = \frac{5}{3} \cdot 60 = 100$. The sum $M + N = 27 + 100 = 127$.

8. **Answer (D):** The sales in the 4 months were \$100, \$60, \$40 and \$120. The average sales were $\dfrac{100 + 60 + 40 + 120}{4} = \dfrac{320}{4} = \$80$.

OR

In terms of the \$20 intervals, the sales were 5, 3, 2 and 6 on the chart. Their sum is $5 + 3 + 2 + 6 = 16$ and the average is $\frac{16}{4} = 4$. The average sales were $4 \cdot \$20 = \$80$.

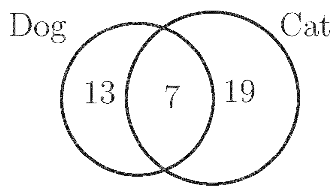
9. **Answer (D):** At the end of the first year, Tammy's investment was 85% of the original amount, or \$85. At the end of the second year, she had 120% of her first year's final amount, or 120% of $\$85 = 1.2(\$85) = \$102$. Over the two-year period, Tammy's investment changed from \$100 to \$102, so she gained 2%.

10. **Answer (D):** The sum of the ages of the 6 people in Room A is $6 \times 40 = 240$. The sum of the ages of the 4 people in Room B is $4 \times 25 = 100$. The sum of the ages of the 10 people in the combined group is $100 + 240 = 340$, so the average age of all the people is $\frac{340}{10} = 34$.

11. **Answer (A):** The number of cat owners plus the number of dog owners is $20 + 26 = 46$. Because there are only 39 students in the class, there are $46 - 39 = 7$ students who have both.

OR

Because each student has at least a cat or a dog, there are $39 - 20 = 19$ students with a cat but no dog, and $39 - 26 = 13$ students with a dog but no cat. So there are $39 - 13 - 19 = 7$ students with both a cat and a dog.



12. **Answer (C):** The table gives the height of each bounce.

Bounce	1	2	3	4	5
Height in Meters	2	$\frac{2}{3} \cdot 2 =$ $\frac{4}{3}$	$\frac{2}{3} \cdot \frac{4}{3} =$ $\frac{8}{9}$	$\frac{2}{3} \cdot \frac{8}{9} =$ $\frac{16}{27}$	$\frac{2}{3} \cdot \frac{16}{27} =$ $\frac{32}{81}$

Because $\frac{16}{27} > \frac{16}{32} = \frac{1}{2}$ and $\frac{32}{81} < \frac{32}{64} = \frac{1}{2}$, the ball first rises to less than 0.5 meters on the fifth bounce.
Note: Because all the fractions have odd denominators, it is easier to double the numerators than to halve the denominators. So compare $\frac{16}{27}$ and $\frac{32}{81}$ to their numerators' fractional equivalents of $\frac{1}{2}$, $\frac{16}{32}$ and $\frac{32}{64}$.

13. **Answer (C):** Because each box is weighed two times, once with each of the other two boxes, the total $122 + 125 + 127 = 374$ pounds is twice the combined weight of the three boxes. The combined weight is $\frac{374}{2} = 187$ pounds.

14. **Answer (C):** There are only two possible spaces for the B in row 1 and only two possible spaces for the A in row 2. Once these are placed, the entries in the remaining spaces are determined.

The four arrangements are:

A	B	C
B	C	A
C	A	B

A	B	C
C	A	B
B	C	A

A	C	B
C	B	A
B	A	C

A	C	B
B	A	C
C	B	A

OR

The As can be placed either

A		
	A	
		A

or

A		
		A
	A	

In each case, the letter next to the top A can be B or C. At that point the rest of the grid is completely determined. So there are $2+2 = 4$ possible arrangements.

15. **Answer (B):** The sum of the points Theresa scored in the first 8 games is 37. After the ninth game, her point total must be a multiple of 9 between 37 and $37 + 9 = 46$, inclusive. The only such point total is $45 = 37 + 8$, so in the ninth game she scored 8 points. Similarly, the next point total must be a multiple of 10 between 45 and $45 + 9 = 54$. The only such point total is $50 = 45 + 5$, so in the tenth game she scored 5 points. The product of the number of points scored in Theresa's ninth and tenth games is $8 \cdot 5 = 40$.

16. **Answer (D):** The volume is $7 \times 1 = 7$ cubic units. Six of the cubes have 5 square faces exposed. The middle cube has no face exposed. So the total surface area of the figure is $5 \times 6 = 30$ square units. The ratio of the volume to the surface area is $7 : 30$.

OR

The volume is $7 \times 1 = 7$ cubic units. There are five unit squares facing each of six directions: front, back, top, bottom, left and right, for a total of 30 square units of surface area. The ratio of the volume to the surface area is $7 : 30$.

17. **Answer (D):** The formula for the perimeter of a rectangle is $2l + 2w$, so $2l + 2w = 50$, and $l + w = 25$. Make a chart of the possible widths, lengths, and areas, assuming all the widths are shorter than all the lengths.

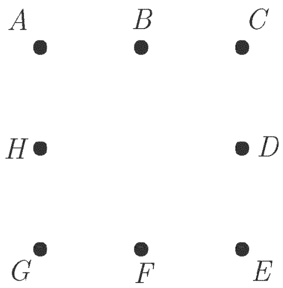
Width	1	2	3	4	5	6	7	8	9	10	11	12
Length	24	23	22	21	20	19	18	17	16	15	14	13
Area	24	46	66	84	100	114	126	136	144	150	154	156

The largest possible area is $13 \times 12 = 156$ and the smallest is $1 \times 24 = 24$, for a difference of $156 - 24 = 132$ square units.
Note: The product of two numbers with a fixed sum increases as the numbers get closer together. That means, given the same perimeter, the square has a larger area than any rectangle, and a rectangle with a shape closest to a square will have a larger area than other rectangles with equal perimeters.

18. **Answer (E):** The length of first leg of the aardvark's trip is $\frac{1}{4}(2\pi \times 20) = 10\pi$ meters. The third and fifth legs are each $\frac{1}{4}(2\pi \times 10) = 5\pi$ meters long. The second and sixth legs are each 10 meters long, and the length of the fourth leg is 20 meters. The length of the total trip is $10\pi + 5\pi + 5\pi + 10 + 10 + 20 = 20\pi + 40$ meters.

19. **Answer (B):** Choose two points. Any of the 8 points can be the first choice, and any of the 7 other points can be the second choice. So there are $8 \times 7 = 56$

ways of choosing the points in order. But each pair of points is counted twice, so there are $\frac{56}{2} = 28$ possible pairs.



Label the eight points as shown. Only segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$ and $\overline{HA}$ are 1 unit long. So 8 of the 28 possible segments are 1 unit long, and the probability that the points are one unit apart is $\frac{8}{28} = \frac{2}{7}$.

OR

Pick the two points, one at a time. No matter how the first point is chosen, exactly 2 of the remaining 7 points are 1 unit from this point. So the probability of the second point being 1 unit from the first is $\frac{2}{7}$.

20. **Answer (B):** Because $\frac{2}{3}$ of the boys passed, the number of boys in the class is a multiple of 3. Because $\frac{3}{4}$ of the girls passed, the number of girls in the class is a multiple of 4. Set up a chart and compare the number of boys who passed with the number of girls who passed to find when they are equal.

Total boys	Boys passed
3	2
6	4
9	6

Total girls	Girls passed
4	3
8	6

The first time the number of boys who passed equals the number of girls who passed is when they are both 6. The minimum possible number of students is $9 + 8 = 17$.

OR

Because $\frac{2}{3}$ of the boys passed, the number of boys who passed must be a multiple of 2. Because $\frac{3}{4}$ of the girls passed, the number of girls who passed must be a multiple of 3. Because the same number of boys and girls passed, the smallest possible number is 6, the least common multiple of 2 and 3. If 6 of 9 boys and 6 of 8 girls passed, there are 17 students in the class, and that is the minimum number possible.

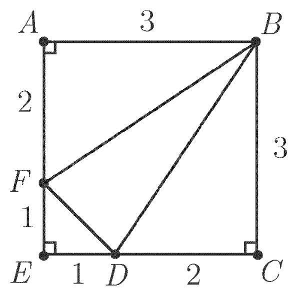
OR

Let $G =$ the number of girls and $B =$ the number of boys. Then $\frac{2}{3}B = \frac{3}{4}G$, so $8B = 9G$. Because 8 and 9 are relatively prime, the minimum number of boys and girls is 9 boys and 8 girls, for a total of $9 + 8 = 17$ students.

21. **Answer (C):** Using the formula for the volume of a cylinder, the bologna has volume $\pi r^2 h = \pi \times 4^2 \times 6 = 96\pi$. The cut divides the bologna in half. The half-cylinder will have volume $\frac{96\pi}{2} = 48\pi \approx 151$ cm^3.
Note: The value of π is slightly greater than 3, so to estimate the volume multiply $48(3) = 144$ cm^3. The product is slightly less than and closer to answer C than any other answer.

22. **Answer (A):** Because $\frac{n}{3}$ is at least 100 and is an integer, n is at least 300 and is a multiple of 3. Because $3n$ is at most 999, n is at most 333. The possible values of n are $300, 303, 306, \ldots, 333 = 3 \cdot 100, 3 \cdot 101, 3 \cdot 102, \ldots, 3 \cdot 111$, so the number of possible values is $111 - 100 + 1 = 12$.

23. **Answer (C):** Because the answer is a ratio, it does not depend on the side length of the square. Let $AF = 2$ and $FE = 1$. That means square $ABCE$ has side length 3 and area $3^2 = 9$ square units. The area of $\triangle BAF$ is equal to the area of $\triangle BCD = \frac{1}{2} \cdot 3 \cdot 2 = 3$ square units. Triangle DEF is an isosceles right triangle with leg lengths $DE = FE = 1$. The area of $\triangle DEF$ is $\frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ square units. The area of $\triangle BFD$ is equal to the area of the square minus the areas of the three right triangles: $9 - (3 + 3 + \frac{1}{2}) = \frac{5}{2}$. So the ratio of the area of $\triangle BFD$ to the area of square $ABCE$ is $\dfrac{\frac{5}{2}}{9} = \dfrac{5}{18}$.



24. **Answer (C):** There are $10 \times 6 = 60$ possible pairs. The squares less than 60 are 1, 4, 9, 16, 25, 36 and 49. The possible pairs with products equal to the given squares are $(1,1)$, $(2,2)$, $(1,4)$, $(4,1)$, $(3,3)$, $(9,1)$, $(4,4)$, $(8,2)$, $(5,5)$, $(6,6)$ and $(9,4)$. So the probability is $\frac{11}{60}$.

25. **Answer (A):**

Circle #	Radius	Area
1	2	4π
2	4	16π
3	6	36π
4	8	64π
5	10	100π
6	12	144π

The total black area is $4\pi + (36-16)\pi + (100-64)\pi = 60\pi$ in^2.
So the percent of the design that is black is $100 \times \frac{60\pi}{144\pi} = 100 \times \frac{5}{12}$ or about 42%.