The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Wednesday, November 25, 2015
(in North America and South America)

Thursday, November 26, 2015
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Do not open this booklet until instructed to do so.
There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.** For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

Canadian Intermediate Mathematics Contest

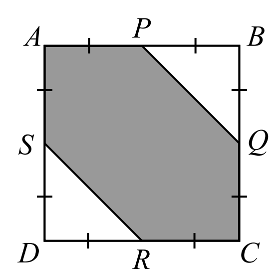
NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

PART A

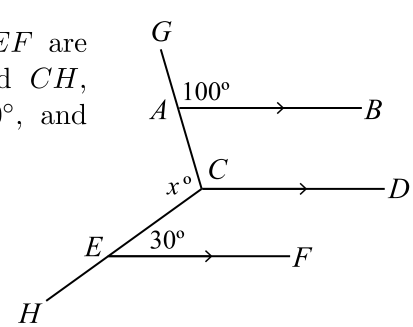
For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. Stephanie has 1000 eggs to pack into cartons of 12 eggs. While packing the eggs, she breaks n eggs. The unbroken eggs completely fill a collection of cartons with no eggs left over. If $n < 12$, what is the value of n?

2. In the diagram, $ABCD$ is a square with side length 4. Points P, Q, R, and S are the midpoints of the sides of the square, as shown. What is the area of the shaded region?



3. In the diagram, line segments AB, CD and EF are parallel, and points A and E lie on CG and CH, respectively. If $\angle GAB = 100^\circ$, $\angle CEF = 30^\circ$, and $\angle ACE = x^\circ$, what is the value of x?



4. If $12x = 4y + 2$, determine the value of the expression $6y - 18x + 7$.

5. Determine the *largest* positive integer n with $n < 500$ for which $6048(28^n)$ is a perfect cube (that is, it is equal to m^3 for some positive integer m).

6. A total of 2015 tickets, numbered $1, 2, 3, 4, \ldots, 2014, 2015$, are placed in an empty bag. Alfie removes ticket a from the bag. Bernice then removes ticket b from the bag. Finally, Charlie removes ticket c from the bag. They notice that $a < b < c$ and $a + b + c = 2018$. In how many ways could this happen?

PART B

For each question in Part B, your solution must be well organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. At Cornthwaite H.S., many students enroll in an after-school arts program. The program offers a drama class and a music class. Each student enrolled in the program is in one class or both classes.

   (a) This year, 41 students are in the drama class and 28 students are in the music class. If 15 students are in both classes, how many students are enrolled in the program?

   (b) In 2014, a total of 80 students enrolled in the program. If $3x - 5$ students were in the drama class, $6x + 13$ students were in the music class, and x students were in both classes, determine the value of x.

   (c) In 2013, half of the students in the drama class were in both classes and one-quarter of the students in the music class were in both classes. A total of N students enrolled in the program in 2013. If N is between 91 and 99, inclusive, determine the value of N.

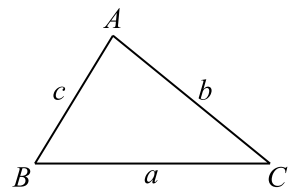
2. Alistair, Conrad, Emma, and Salma compete in a three-sport race. They each swim 2 km, then bike 40 km, and finally run 10 km. Also, they each switch instantly from swimming to biking and from biking to running.

   (a) Emma has completed $\frac{1}{13}$ of the total distance of the race. How many kilometers has she travelled?

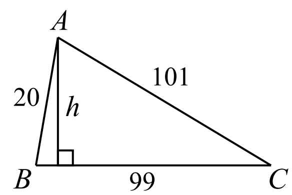
   (b) Conrad began the race at 8:00 a.m. and completed the swimming portion in 30 minutes. Conrad biked 12 times as fast as he swam, and ran 3 times as fast as he swam. At what time did he finish the race?

   (c) Alistair and Salma also began the race at 8:00 a.m. Alistair finished the swimming portion in 36 minutes, and then biked at 28 km/h. Salma finished the swimming portion in 30 minutes, and then biked at 24 km/h. Alistair passed Salma during the bike portion. At what time did Alistair pass Salma?

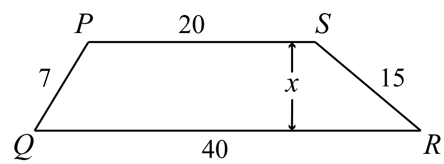
3. Heron's Formula says that if a triangle has side lengths a, b and c, then its area equals $\sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$ is called the *semi-perimeter* of the triangle.



(a) In the diagram, $\triangle ABC$ has side lengths $AB = 20$, $BC = 99$, and $AC = 101$. If h is the perpendicular distance from A to BC, determine the value of h.



(b) In the diagram, trapezoid $PQRS$ has PS parallel to QR. Also, $PQ = 7$, $QR = 40$, $RS = 15$, and $PS = 20$. If x is the distance between parallel sides PS and QR, determine the value of x.



(c) The triangle with side lengths 3, 4 and 5 has the following five properties:

- its side lengths are integers,
- the lengths of its two shortest sides differ by one,
- the length of its longest side and the semi-perimeter differ by one,
- its area is an integer, and
- its perimeter is less than 200.

Determine all triangles that have these five properties.